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STATE OF STRESS OF NONUNIFORMLY HEATED PLATES LOADED ALONG THE BOUNDARY SURFACES

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The general solution of an elasticity theory problem for a constant thickness plate is constructed under the condition that a force and a nonuniformly heated plate are applied normally to the boundary planes. The solution is obtained as a result of applying the M. E. Vashchenko-Zakharchenko expansion formulas to the infinitely high-order differential equations obtained by A. I. Lur'e by a symbolic method [1, 2], by a separate analysis of the symmetric and anti-symmetric elasticity theory problems relative to the middle plane: 1) for constant temperature and given forces on the boundary planes; 2) for a given nonuniform heating and no forces. Simple formulas are presented to determine the state of stress in the case of a slowly varying external load and temperature of the unbounded plate. For a bounded plate the general solution for no forces on the boundary planes and heating resulted in the A. I. Lur'e solution [1].

1. Let the stresses

$$\tau_{zx} = \tau_{zy} = 0, \quad \sigma_{zz} = \sigma_{\pm}, \quad z = \pm h$$

be given on the boundary planes of a uniformly heated layer of thickness $2h$.

Let us solve the symmetric and antisymmetric problems separately by using the representation $\sigma_{\pm} = q \pm p$.

1°. In the symmetric case $(p = 0)$, on the basis of [2] the elastic strain state can be represented in the form

$$u = -\frac{\partial}{\partial x} f_1(d, \zeta)\varkappa, \quad w = -\frac{d^2}{h} f_2(d, \zeta)\varkappa \tag{1.1}$$

$$\sigma_{xx} = 2G\left[-\frac{\partial^2}{\partial x^2} f_1(d, \zeta) + \frac{2\nu}{h^2} d^2 f_0(d, \zeta)\right]\varkappa, \ \sigma_{zz} = 2G\frac{d^2}{h^2} f_3(d, \zeta)\varkappa$$

$$f_0(d, \zeta) = \cos d\zeta \sin d/d, \ f_1(d, \zeta) = g(d, \zeta) - (1 - 2\nu) f_0(d, \zeta)$$

$$f_2(d, \zeta) = [-d \cos d \sin d\zeta + d\zeta \cos d\zeta \sin d - 2(1-\nu) \times \sin d\zeta \sin d] d^{-2}$$

$$f_3(d, \zeta) = g(d, \zeta) + f_0(d, \zeta), \quad g(d, \zeta) = \cos d\zeta \cos d + \zeta \sin d\zeta \sin d$$

$$\zeta = \frac{z}{h}, \quad \varkappa = \frac{h^2}{2G} \frac{q}{d^2(1 + \sin 2d / 2d)}, \quad d^2 = h^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$$

The displacement v and the stress σ_{yy} can be obtained here and below from the relationships for u and σ_{xx} by replacing $\partial / \partial x$ by $\partial / \partial y$ and $\partial / \partial y$ by $\partial / \partial x$. Other components of the stress tensor can be found on the basis of the u, v, w.

Functions of the form $\Phi = f(d)\,\varkappa$, entering into formulas (1.1) where $f(d)$ is an entire function of arguments d and d^2. It is seen that the function Φ is the solution of an infinitely high order differential equation

$$d^2 (1 + \sin 2d / 2d)\, \Phi = h^2 f(d) q / 2G \qquad (1.2)$$

We apply the M. E. Vashchenko-Zakharchenko expansion formulas [3, 4] to solve (1.2). We then obtain for the functions $f = f_j(d, \zeta)$ $(j = 0, \ldots, 3)$

$$\frac{f(d)\, q}{d^2 (1 + \sin 2d / 2d)} = \sum_i \frac{f(\alpha_i)\, q_i}{\cos^2 \alpha_i} + \frac{f(0)\, q_0}{2} \qquad (1.3)$$

Here α_i are the roots of the algebraic equation $1 + \sin 2\alpha / 2\alpha = 0$, where $\operatorname{Re} \alpha_i > 0$, and the functions q_i are the solutions of the second-order equation $(d^2 - \alpha_i^2) q_i = q$ for $i \neq 0$ and $d^2 q_0 = q$.

Let us furthermore consider just the case of an unbounded plate. In the case the functions q_k $(k = 0, 1, \ldots)$ are determined uniquely from the additional condition that $q_i \to 0$ $(i \neq 0)$, $\partial q_0 / \partial x \to 0$, $\partial q_0 / \partial y \to 0$ for $x, y \to \infty$.

Using (1.3), we obtain

$$u = -\frac{h^2}{2G} \frac{\partial}{\partial x} (Q_1 + \nu q_0), \quad w = -\frac{h}{2G} [d^2 Q_3 - (1 - \nu) \zeta q] \qquad (1.4)$$

$$\sigma_{xx} = -h^2 \frac{\partial^2}{\partial x^2} Q_1 + 2\nu d^2 Q_0 + \nu h^2 \frac{\partial^2}{\partial y^2} q_0, \quad \sigma_{zz} = d^2 Q_3 + q$$

$$Q_j = \sum \frac{f_j(\alpha_i, \zeta)\, q_i}{\cos^2 \alpha_i}$$

It is seen that the series converge for bounded functions q.

Let $q(x, y)$ be a function varying slowly as a function of x / h, y / h (or h a small quantity), which has continuous second derivatives. Then $q_i \approx -q / \alpha_i^2$ is an approximate solution of the equation $(d^2 - \alpha_i^2) q_i = q$. Taking accont of the formula

$$\sum_i \frac{f(\alpha_i)}{\alpha_i^2 \cos^2 \alpha_i} = -\left(\frac{f}{6} + \frac{f''}{4}\right)\Big|_{d=0}$$

(which follows from (1.3) if we formally set $d = 0$, $f = 1$), we obtain, say

$$Q_1 = \nu h^2 \frac{\partial^2 q}{\partial y^2} \frac{\zeta^2 - 1/3}{2} + Q_1^*, \quad Q_3 = Q_3^*$$

$$Q_j^* = \sum_i \frac{f_j(\alpha_i, \zeta)(q_i + q / \alpha_i^2)}{\cos^2 \alpha_i}$$

We hence find

$$\sigma_{xx} = \nu h^2 \frac{\partial^2}{\partial y^2} q_0 - \frac{h^2}{2}\left(\zeta^2 - \frac{1}{3}\right)\left(\frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2}\right) q +$$
$$d^2 (-Q_1^* + 2\nu Q_0^*) + h^2 \frac{\partial^2}{\partial y^2} Q_1^*$$

The series for Q_j^* will hence converge more rapidly. It is possible to set $Q_j^* \approx 0$ approximately for very slowly varying functions q.

A more exact (asymptotic) solution can be constructed also for sufficiently smooth and slowly varying functions q by setting $q \approx -q / \alpha_i^2 - d^2 q / \alpha_i^4 - \ldots$ and summing the series thus originating. Such approximate expressions for Q_j are also obtained by a direct expansion of the functions

$$\left[\frac{f_j(d, \zeta)}{d^2(1 + \sin 2d/2d)} - \frac{f_j(0, \zeta)}{2d^2}\right] q$$

in a series of powers of d.

Now, let q not be a slowly varying function. The approximate expression written down for q_i will be valid (for sufficiently smooth functions $q(x, y)$ even in this case, but only for large numbers i. Let such a solution be sufficiently exact for $i \geqslant m$. The asymptotic expression for Q_j will then have the form

$$Q_j = \sum_{i=1}^{n-1} \frac{f_j(\alpha_i, \zeta)\, q_i}{\cos^2 \alpha_i} + (a_{0j} + a_{2j} d^2 + \ldots)\, q$$

Here a_{kj} are coefficients of a power series expansion in d for the functions

$$\frac{f_j(d, \zeta)}{d^2(1 + \sin 2d/2d)} - \frac{f_j(0, \zeta)}{2d^2} - \sum_{i=1}^{m-1} \frac{f_j(\alpha_i, \zeta)}{\cos^2 \alpha_i} \frac{1}{d^2 - \alpha_i^2}$$

This method of obtaining an approximate solution is analogous to the usual method of improving the convergence of series.

Now, let us estimate the convergence of the series in terms of which the solution is expressed. To do this, we represent the functions q_i in the form $(K_0(x)$ is the Macdonald function, and D is the domain $-\infty < x_0, y_0 < \infty)$

$$q_i(x, y) = -\frac{1}{2\pi} \int_D q(x_0, y_0) K_0(\alpha_i r)\, dx_0 dy_0 \quad \left(r = \sqrt{(x - x_0)^2 + (y - y_0)^2}\right) \qquad (1.5)$$

which can be written thus

$$q_i(x, y) = -\frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} q_*(x, y; \eta, \varphi) K_0(\alpha_i \eta)\, \eta\, d\varphi\, d\eta \qquad (1.6)$$

$$q_*(x, y; \eta, \varphi) = q(x + \eta \cos \varphi,\ y + \eta \sin \varphi)$$

Let us separate the integral over η into two integrals, from 0 to l and from l to ∞, by selecting the constant l so that the function q_* would be continuous together with its two first derivatives with respect to η for $\eta < l$. Let us also assume that q is a function bounded in D. We represent the function q_* in the section $(0, l)$ as a Maclauren series in the variable η

$$q_*(x, y; \eta, \varphi) = q(x, y) + \eta \left.\frac{\partial q_*}{\partial \eta}\right|_{\eta=0} + \frac{\eta^2}{2} \left.\frac{\partial^2 q_*}{\partial \eta^2}\right|_{\eta=\eta_*}$$

$$0 \leqslant \eta_* \leqslant l$$

and we use an asymptotic representation in the section (l, ∞)

$$K_0(x) \approx \sqrt{\frac{\pi}{2x}}\, e^{-x}\left(1 - \frac{1}{8x} + \ldots\right), \quad x \gg 1$$

We consequently have a representation for the functions q_n for large n

$$q_n(x, y) = -q / \alpha_n^2 + O(\alpha_n^{-4}) + O(\exp(-\alpha_n l)) \qquad (1.7)$$

Let us emphasize that the point (x, y) belongs to that domain D_0 in which $q(x, y)$ is a function continuously twice differentiable with respect to x and y while l is the distance between the point (x, y) and the boundary of the domain D_0.

Taking (1.7) into account, as well as the estimates $\alpha_n = O(n)$, $\exp(i\zeta\alpha_n) = O(\alpha_n^{\zeta/2})$, we see that both the series in which the functions Q_j and Q_j^* are expressed, and the series obtained by differentiating them term-by-term with respect to x and y, will converge. The series in which the functions Q_j^* are expressed can be differentiated twice term-by-term with respect to x and y. Therefore, the solution expressed in terms of the functions Q_j^* is more convenient, as has already been mentioned above. The terms of the series for the displacements, obtained after term-by-term differentiation, will hence be of the order of n^{-3} for large n, while the terms in the series for the stresses will be of the order of n^{-2}.

A solution in still more rapidly converging series can be obtained for smoother functions q. Thus, if the function q has four continuous derivatives with respect to x and y in the domain D_0, then we obtain in place of (1.7)

$$q_n(x, y) = -q / \alpha_n^2 - d^2 q / \alpha_n^4 + O(\alpha_n^{-6}) + O(\exp(-\alpha_n l)) \quad (x, y) \in D_0$$

Proceeding as above, i.e., summing the series and introducing the functions

$$Q_j^{**} = \sum_i \left(g_i + \frac{q}{\alpha_i^2} + \frac{d^2 q}{\alpha_i^4}\right) \frac{f_j(\alpha_i \zeta)}{\cos^2 \alpha_i}$$

we obtain a series for the displacements, whose terms have the order n^{-5} for large n, and the order n^{-4} for the stresses. The series converge considerably more rapidly in the domain D_c in which $q = \text{const}$. Then on the basis of (1.4) and (1.5) and with the asumptotic representation for the function $K_0(x)$ taken into account for larger x, we see that the terms of the series in which the displacements and stresses are defined in the domain D_c will be of the order $\exp(-\alpha_n \rho)$ for larger n (ρ is the distance between the point (x, y) and the boundary of the domain D_c).

2°. In the antisymmetric case $(q \equiv 0)$, the elastic state of strain of a plate can be represented in the form

$$u = -\frac{h^2}{2G}\frac{\partial}{\partial x}(P_1 + \omega_1), \quad w = \frac{h}{2G}(P_2 + \omega_2) \qquad (1.8)$$

$$\sigma_{xx} = -h^2 \frac{\partial^2}{\partial x^2}(P_1 + \omega_1) + 2\nu d^2 (P_0 + \omega_0)$$

$$\sigma_{zz} = d^2 P_3 + \frac{3}{2}\left(\zeta - \frac{\zeta^3}{3}\right) p$$

$$P_j = \sum_i \frac{\varphi_j(\beta_i, \zeta)\, p_i}{\sin^2 \beta_i}, \quad \omega_0 = -\frac{3}{2}\zeta p_0 + \frac{3}{2}\left(0{,}3\zeta + \frac{\zeta^3}{6}\right) d^2 p_0$$

$$\omega_1 = 3(1-\nu)\zeta p_0 + [0.5(\nu-2)\zeta^3 + 0.3(2+3\nu)\zeta] d^2 p_0$$
$$\omega_2 = 3(1-\nu)p_0 + [1{,}5\nu\zeta^2 - 0{,}3(8-3\nu)] d^2 p_0$$
$$\varphi_0(d,\ \zeta) = -\cos d \sin d\zeta / d, \quad \varphi_1(d,\ \zeta) = (1-2\nu)\varphi_0(d,\ \zeta) + \psi(d,\ \zeta)$$
$$\varphi_2(d,\ \zeta) = 2(1-\nu)\cos d\zeta \cos d + d\,\zeta \sin d\zeta \cos d - d \sin d \cos d\zeta$$
$$\varphi_3(d,\ \zeta) = \psi(d,\ \zeta) + \varphi_0(d,\ \zeta), \quad \psi(d,\ \zeta) = \zeta \cos d\zeta \cos d + \sin d\zeta \sin d$$

Here the functions p_i are determined from the solution of the equation $(d^2 - \beta_i^2)p_i = p$ for $i \neq 0$, $d^4 p_0 = p$, β_i are the roots of the equation $1 - \sin 2\beta / 2\beta = 0$, where $\operatorname{Re} \beta_i > 0$.

The solution of (1.8) is obtained by a method analogous to that used in section 1.1°.

We see from (1.8) that the equation for the plate deflections can be written in the form

$$w = w_0 + w_* \tag{1.9}$$
$$w_* = \frac{h}{2G} P_2, \quad d^4 w_0 = \frac{h}{2G}\{3(1-\nu)p + [1.5\nu\zeta^2 - 0.3(8-3\nu)] d^2 p\}$$

which is a generalization of the equation of medium-thickness plate deflections [5] found by an approximate method, where $w_* = 0$ is hence obtained.

If p is a slowly varying function of the variables x/h and y/h, then $p_i \approx -p/\beta_i^2$. Summing the series, as above, we obtain for $\zeta = 1$

$$P_1 = 0.063(1-\nu)p + P_1^*, \quad P_2 = 0.246(1-\nu)p + P_2^*$$
$$P_j^* = \sum_i \left(p_i + \frac{p}{\beta_i^2}\right) \frac{\varphi_j(\beta_i, \zeta)}{2\sin^2 \beta_i}$$

We have $P_j^* \approx 0$ for very slowly varying functions. The convergence of the series can be investigated exactly as in Sect. 1.1°.

2. Let us consider the problem of determining the thermoelastic state of a layer under nonuniform heating conditions. The solution of such a problem is executed by a known method [2]: the particular solution of the thermoelasticity equations is first determined without satisfying all the given (zero) boundary conditions, then the correcting solution of the isothermal problem is found.

The components of the displacement vector and the stress tensor which correspond to the particular solution can be represented in the form [2]

$$u = \frac{\partial F}{\partial x}, \quad w = \frac{\partial F}{\partial z}, \quad \tau_{zx} = 2G \frac{\partial}{\partial x}\left(\frac{\partial F}{\partial z}\right)$$
$$\sigma_{xx} = 2G\left(\frac{\partial^2 F}{\partial x^2} - \frac{1+\nu}{1-\nu}\alpha_T t\right), \quad \sigma_{zz} = -2G \frac{d^2}{h^2} F$$

Here t is the layer temperature, α_T is the coefficient of linear expansion,and the function F is determined from the equation

$$\Delta F = \frac{1+\nu}{1-\nu}\alpha_T t, \quad \Delta = \frac{\partial^2}{\partial z^2} + \frac{d^2}{h^2} \tag{2.1}$$

1°. Let the layer temperature t be a symmetric function relative to the plane $z = 0$. The the function t can be represented in the form

$$t = \frac{1}{h}\sum_{n=0}^{\infty} \varepsilon_n t_n \cos \mu_n \zeta \tag{2.2}$$

$$\varepsilon_n = 1 \quad \text{for} \quad n \geqslant 1, \ \varepsilon_0 = 0.5, \ \mu_n = \pi n, \ t_n = \int_{-h}^{h} t \cos \mu_n \zeta \, dz$$

The solution of (2.1) satisfying the conditions $\tau_{zx} = \tau_{zy} = 0$ for $z = \pm h$ will then be

$$F = \frac{1+\nu}{1-\nu}\alpha_T h \sum_{n=0}^{\infty} \varepsilon_n \frac{t_n}{d^2 - \mu_n^2} \cos \mu_n \zeta \tag{2.3}$$

Here $1 / (d^2 - \mu_n^2)$ is an operator inverse to the operator $d^2 - \mu_n^2$, i.e., if the function $t_n / (d^2 - \mu_n^2)$ is denoted by t_n^*, then t_n^* is determined from the equation $(d^2 - \mu_n^2) t_n^* = t_n$.

We hence see that the stresses corresponding to the correcting solution should satisfy the conditions

$$\tau_{zx} = \tau_{zy} = 0, \quad \sigma_{zz} = \sigma_0, \quad z = \pm h$$

$$\sigma_0 = 2G\frac{1+\nu}{1-\nu}\alpha_T \frac{d^2}{h}\sum_{n=0}^{\infty}(-1)^n \varepsilon_n \frac{t_n}{d^2 - \mu_n^2}$$

The problem of determining the correcting solution therefore reduces to the problem from Sect. 1 for $q = \sigma_0$.

We henceforth limit ourselves to finding just a particular solution of the equations which occur. The solution obtained will be general only for an infinite layer. The functions $f(d)\varkappa$ in the formulas for the displacements and stresses will be represented in the form

$$f(d)\varkappa = \frac{1+\nu}{1-\nu}\alpha_T h \left[\sum_i b_i f(\alpha_i) \frac{\tau_i}{d^2 - \alpha_i^2} + \right. \tag{2.4}$$

$$\left. \sum_{n=1}^{\infty} (-1)^n f(\mu_n) \frac{t_n}{d^2 - \mu_n^2} + \frac{f(0)}{4}\frac{t_0}{d^2}\right], \quad b_i = -\frac{1}{2\cos\alpha_i}$$

by using the expansion formulas and taking into account that

$$\sum_{n=0}^{\infty}(-1)^n \varepsilon_n \frac{t_n}{\alpha_i^2 - \mu_n^2} = \frac{\tau_i}{2\alpha_i \sin\alpha_i}$$

$$\tau_i = \int_{-h}^{h} t \cos \alpha_i \zeta \, dz$$

Adding the displacements and stresses corresponding to the particular and correcting solutions we obtain

$$u = -\frac{1+\nu}{1-\nu}\alpha_T h \frac{\partial}{\partial x}\left(T_1 - \frac{1-\nu}{2}\frac{t_0}{d^2}\right) \tag{2.5}$$

$$w = \frac{1+\nu}{1-\nu}\alpha_T h\left(-\frac{d^2}{h^2}T_2 + \int_0^\zeta t\,d\zeta - \frac{\nu\zeta}{2h}t_0\right)$$

$$\sigma_{xx} = 2G\frac{1+\nu}{1-\nu}\alpha_T h\left(-\frac{\partial^2 T_1}{\partial x^2} + 2\nu\frac{d^2}{h^2}T_0 - \frac{1-\nu}{2}\frac{\partial^2}{\partial y^2}\frac{t_0}{d^2} + \frac{t_0}{2h^2} - \frac{t}{h}\right)$$

$$\sigma_{zz} = 2G\frac{1+\nu}{1-\nu}\frac{\alpha_T}{h}d^2 T_3, \quad T_j = \sum_i b_i f_j(\alpha_i, \zeta)\frac{\tau_i}{d^2 - \alpha_i^2}$$

Let us consider the case when t (meaning also the functions τ_i) varies slowly as a function of the variables x / h and y / h. Then $\tau_i / (d^2 - \alpha_i^2) \approx -\tau_i / \alpha_i^2$. In this case the functions T_j can be represented in simple form. Let us limit ourselves to finding the quantity σ_{xx} for $\zeta = 1$. We have

$$-\frac{\partial^2}{\partial x^2}T_1 + 2\nu\frac{d^2}{h^2}T_0 = -2\frac{d^2}{h^2}B$$

$$B = \sum_i b_i \frac{\tau_i}{d^2 - \alpha_i^2} \approx \frac{1}{24}\int_{-h}^{h} t\,(1 + 3\zeta^2)\,dz$$

We have here used the relationship $(f = f_j,\ j = 0, \ldots, 3)$

$$\sum_i \frac{f(\alpha_i)}{\alpha_i \sin\alpha_i(1 + \cos 2\alpha_i)} = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{f(\mu_n)}{\mu_n^2} + \frac{1}{8}[f(0) + f''(0)]$$

Therefore, for a slowly varying function t for $\zeta = 1$

$$\sigma_{xx} \approx 2G\frac{1+\nu}{1-\nu}\alpha_T h\left[-\frac{1-\nu}{2}\frac{\partial^2}{\partial y^2}\frac{t_0}{d^2} + \frac{t_0}{2h^2} - \frac{t}{h}\bigg|_{\zeta=1} - \frac{1}{12}\left(\frac{\partial^2}{\partial x^2} + \nu\frac{\partial^2}{\partial y^2}\right)\int_{-h}^{h} t\,(1 + 3\zeta^2)\,dz\right]$$

2°. The solution for the case when the temperature is an antisymmetric function relative to $z = 0$. Without derivation, let us just present the formulas for the displacements and stresses

$$u = -\frac{1+\nu}{1-\nu}\alpha_T h\frac{\partial}{\partial x}T_{1,1} + \frac{3}{2}(1+\nu)\zeta\frac{\partial}{\partial x}\frac{\tau_{0,1}}{d^2} \tag{2.6}$$

$$w = \frac{1+\nu}{1-\nu}\alpha_T h T_{2,1} - \frac{3}{2}(1+\nu)\frac{\alpha_T}{h}\frac{\tau_{0,1}}{d^2}$$

$$\sigma_{xx} = 2G\frac{1+\nu}{1-\nu}\alpha_T h\left[-\frac{\partial^2}{\partial x^2}T_{1,1} + 2\nu\frac{d^2}{h^2}T_{0,1} - \frac{t}{h} + \frac{3}{2}\frac{\zeta}{h^3}\tau_{0,1} - \frac{3}{2h}(1-\nu)\zeta\frac{\partial^2}{\partial y^2}\frac{\tau_{0,1}}{d^2}\right]$$

$$T_{j,1}=\sum_i b_{i,1}\varphi_j(\beta_j,\zeta)\frac{\tau_{i,1}}{d^2-\beta_i^2};\quad \tau_{i,1}=\int_{-h}^{h} t\sin\beta_i\zeta\,dz,\quad i\neq 0$$

$$\tau_{0,1}=\int_{-h}^{h} tz\,dz,\quad b_{i,1}=-\frac{1}{2\sin\beta_i}$$

3. Let us consider a bounded plate occupying a domain Ω. The boundary consists of surfaces $z=\pm h$ and the cylindrical surface Γ whose equation has the form $\gamma(x,y)=0$, $|z|<h$. The thermoelasticity problem for such a plate for forces given on the planes $z=\pm h$ can be reduced by ordinary means to the solution of the elasticity theory problem for an unheated plate without stresses on the boundary planes, i.e., to the A. I. Lur'e problem. Additional components determined from the solution of the thermoelasticity problem for an infinite layer in which the temperature and stress are the same for $z=\pm h$ in the domain Ω as they would be for a finite plane while the temperature and stress for $z=\pm h$ outside the domain Ω can be selected arbitrarily, are hence added to the displacements or stresses given on the surface Γ. We call such a problem fundamental. The difficulty occurring in such a method of solution is to find a sufficiently simple solution of the thermoelasticity problem for an infinite layer.

Let us note that the fundamental problem has a simple solution besides the known cases ($p=$ const, $q=$ const, $t=$ const, loading a slab with cavities at infinity, etc.) also when the temperature field of the plate is stationary. This permitted the authors of [6, 7] to obtain the effective solution of a number of thermoelasticity problems for bounded slabs and slabs with cavities. Let us note that the solutions obtained herein in Sects. 1 and 2 can be used to find the solution of the fundamental problem.

Another means of solving the problem is possible, without the preliminary determination of the elastic state of strain of the fundamental problem. To do this we generalize the A. I. Lur'e formula [1] to the case when $t\neq 0$, $p\neq 0$, $q\neq 0$. We assume first that stresses or displacements are given on the surface Γ, while forces are applied normally on the planes $z=\pm h$. The solution of such a problem can again be obtained by a separate analysis of corresponding symmetric and antisymmetric problems and their subsequent addition.

In this case the solution of the symmetric problem ($\tau_{zx}=\tau_{zy}=0, \sigma_{zz}=q$ for $z=\pm h$) is composed of three terms. The first is given by (1.4) in which it is only known that the functions q_i are the solutions of the appropriate differential equations. The two other terms (which vanish in the case of an infinite plate) are the biharmonic and vortex state of stress and strain [1, 8, 9]. The solution in the antisymmetric case is comprised of two terms. The first is given by (1.8), and the second by the vortex state of stress and strain [1, 8, 9].

We obtain a representation of the solution for a bounded slab in the case of nonuniform heating by an analogous means. The general solution is composed of the solutions (2.5) and (2.6) as well as the biharmonic and vortex states of stress presented in [1, 8, 9]. The solution obtained herein for $p=q=t=0$ can be reduced to the form given in [1, 8, 9].

Methods developed in [6 — 10] can be used with insignificant changes to solve specific problems on determining the state of stress of nonuniformly heated plates loaded on the boundary planes.

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